

125.340 Formula Sheet

NOTE:

- There are formulas here you may not need. They start from extremely simple to considerably more complex.
- Do not assume every formula corresponds to a test question.
- Labels and subscripts mostly have been removed; these are only intended to help you with details of formulas you have “almost” memorized.

- 1 $PV = \frac{FV}{(1+i)^n}$ present value, worth today, lump sum, single future amount, discount back
- 2 $PV = FV \left(e^{-i^*t} \right)$ PV with continuous compounding, continuously compounded rate, e
- 3 $FV = PV (1+i)^n$ future value, grow to, compounded annually, lump sum invested today
- 4 $FV = PV \left(e^{i^*t} \right)$ FV with continuous compounding, continuously compounded rate, e
- 5 $PV = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \dots + \frac{CF_n}{(1+i)^n}$ uneven / different cash flows each year, cash flow stream, project cash flows, NPV
- 6 $PV = \frac{CF}{i} - \frac{CF/i}{(1+i)^n}$ annuity as perpetuity minus delayed perpetuity, perpetuity (CF/i), forever
- 7 $PV = CF \left[\frac{1 - (1+i)^{-n}}{i} \right]$ PV of annuity, equal payments each period, loan repayment, mortgage, pension
- 8 $FV = \frac{CF}{i} (1+i)^n - \frac{CF}{i}$ FV of annuity (perpetuity form), regular savings, accumulate
- 9 $FV = CF \left[\frac{(1+i)^n - 1}{i} \right]$ FV of annuity, saving each year, regular deposits, how much will you have
- 10 $APR = MonthlyRate * 12$ APR, annual percentage rate, quoted / nominal rate, monthly rate
- 11 $1 + EAR = \left(1 + \frac{APR}{n} \right)^n$ effective annual rate, EAR, compounded monthly / quarterly, true annual rate
- 12 $E(r) = \sum_{s=1}^n p(s)r(s) = \frac{1}{n} \sum_{s=1}^n r(s)$ expected return, scenarios, states of economy, probability, mean / average return
- 13 $g = TV^{1/n} - 1$ geometric average, CAGR, compound annual growth, time-weighted return
- 14 $TV_n = (1+r_1)(1+r_2)\dots(1+r_n)$ terminal value, value of \$1 invested, holding period return over several years

- 15 $\sigma_x^2 = \sum_{i=1}^n p_i (x_i - \bar{x})^2$ variance with probabilities, scenarios, standard deviation, volatility, total risk
- 16 $\sigma_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$ variance from historical data, equally weighted, standard deviation
- 17 $\hat{\sigma}^2 = \frac{1}{n} \sum_{s=1}^n [r(s) - \bar{r}]^2$ variance of returns, historical sample, standard deviation estimate
- 18 $\bar{x} = \sum_{i=1}^n p_i x_i$ mean with probabilities, expected value
- 19 $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ mean of historical data, arithmetic average
- 20 $\hat{S} = \frac{1}{N} \sum_{t=1}^N \frac{(x_t - \bar{x})^3}{\hat{\sigma}^3}$ skewness, asymmetry, left / right tail, crash risk
- 21 $\hat{K} = \frac{1}{N} \sum_{t=1}^N \frac{(x_t - \bar{x})^4}{\hat{\sigma}^4}$ kurtosis, fat tails, excess kurtosis (minus 3), extreme outcomes, normality
- 22 $U = E(r) - \frac{1}{2} A \sigma^2$ utility, risk aversion A, which portfolio does the investor prefer, certainty equivalent
- 23 $E(r_p) = w_x E(r_x) + w_y E(r_y)$ portfolio expected return, weights, x% in A and rest in B
- 24 $\sigma_p^2 = w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + 2w_x w_y \sigma_{x,y}$ two-asset portfolio risk / standard deviation, diversification, covariance
- 25 $\sigma_p^2 = w_x^2 \sigma_x^2 + w_y^2 \sigma_y^2 + w_z^2 \sigma_z^2 + 2w_x w_y \sigma_{x,y} + 2w_x w_z \sigma_{x,z} + 2w_y w_z \sigma_{y,z}$ three-asset portfolio variance
- 26 $\sigma_p^2 = W * \Sigma * W^T$ covariance matrix, variance-covariance matrix, many assets, Excel MMULT
- 27 $\sigma_{x,y} = \sum_{i=1}^n p_i (x_i - \bar{x})(y_i - \bar{y})$ covariance with probabilities, joint scenarios, co-movement
- 28 $\sigma_{x,y} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$ covariance from historical data, move together
- 29 $\text{Cov}(r_x, r_y) = \sigma_{x,y} = \rho_{x,y} \sigma_x \sigma_y$ convert correlation to covariance
- 30 $\rho_{x,y} = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}$ correlation coefficient, perfectly negative / positive correlation, uncorrelated
- Optimal Allocation to P*
- 31 $y = \frac{E(r_P) - r_f}{A \sigma_P^2}$ optimal allocation / proportion in risky portfolio, y*, risky vs risk-free split, complete portfolio
- 32 $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$ single-index model, index model, alpha, beta, residual, characteristic line
- 33 $R_{i,t}^* = \alpha_i + \beta_i R_{m,t}^* + e_{i,t}$ index model regression of excess returns, security characteristic line, intercept / slope

34	$E(R_i) = \alpha_i + \beta_i E(R_m)$	expected return under index model, forecast with alpha and beta
35	$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$	firm-specific / residual / unsystematic / idiosyncratic / diversifiable risk, systematic risk, R-squared
36	$\sigma_{i,j} = \beta_i \beta_j \sigma_m^2$	covariance between two stocks via index model, correlation through the market
37	$\alpha_p = \sum_{i=1}^n w_i \alpha_i$	portfolio alpha, weighted average alpha
38	$\beta_p = \sum_{i=1}^n w_i \beta_i$	portfolio beta, weighted average beta
39	$\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2}$	calculate beta, covariance with market over market variance, regression slope
40	Adjusted beta = $\frac{2}{3} \times \text{Estimated beta} + \frac{1}{3} \times 1.00$	adjusted beta, Blume / Merrill Lynch, beta mean-reverts towards 1, forecast beta
41	SML: $E(r_i) = r_f + E(r_m) - r_f \beta_i$	CAPM, required return, fair return, cost of equity, under / overpriced, alpha vs SML
42	CAL: $E(r_C) = r_f + \frac{\sigma_C}{\sigma_P} [E(r_P) - r_f]$	capital allocation line, risky portfolio + T-bills, Sharpe ratio, reward-to-volatility
43	CML: $E(r_C) = r_f + \frac{\sigma_C}{\sigma_m} [E(r_m) - r_f]$	capital market line, market portfolio, passive strategy, efficient portfolios
44	$E(r_P) = \sum_k w_k E(r_k)$	portfolio expected return, weighted average
45	$\beta_P = \sum_k w_k \beta_k$	portfolio beta under CAPM, target beta, adding a stock
46	$\tilde{R}_i^* = \alpha_i + \beta_i \tilde{R}_m^* + \varepsilon_i$	index model in excess returns, alpha, beta, residual
47	$R_i = E(R_i) + \beta_{i,F_1} F_1 + \beta_{i,F_2} F_2 + \beta_{i,F_3} F_3 + e_i$	APT, multifactor model, factor betas, factor surprises, macroeconomic factors
48	$R_{it} = \alpha_i + \beta_{i,M} R_{Mt} + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + e_{it}$	Fama-French three-factor, SMB size, HML value, book-to-market
49	$d_t = r_{t,p} - r_{t,Tbill}$	excess return over T-bill
50	$Sharpe = \frac{\bar{d}}{\sigma_d}$	Sharpe ratio, reward to total risk, excess return per unit of standard deviation
51	$T = \frac{R_p - r_f}{\beta_p}$	Treynor measure, reward to systematic risk, excess return per unit of beta
52	$J = R_p - [r_f + (r_m - r_f)\beta]$	Jensen's alpha, abnormal return, did the manager add value, vs CAPM

- 53 $M^2 = r_{p*} - r_M$ M-squared, M2, risk-adjusted return, match market volatility with T-bills
- 54 $d = r_{p,t} - r_{b,t}$ active return, return over benchmark
- 55 $IR = \frac{\bar{d}}{\sigma_d}$ information ratio, appraisal ratio, active return per unit of tracking error
- 56 $TE = \sigma_d$ tracking error, volatility of active return
- 57 $g = ROE \times b$ sustainable growth, retention / plowback ratio, payout ratio, ROE
- 58 $P = C \left[\frac{1 - (1+i)^{-n}}{i} \right] + \frac{Par}{(1+i)^n}$ bond price, coupon, par / face value, yield to maturity, premium / discount bond
- 59 $P = \frac{D_0(1+g)}{k-g}$ dividend discount model, Gordon growth, constant growth, D0 just paid, intrinsic value
- 60 $P = \frac{D}{k}$ no growth, constant dividend forever, preferred stock, perpetuity
- 61 $P = PE \times E$ P/E ratio, price-earnings multiple, relative valuation, comparables, EPS
- 62 $Working\ capital = current\ assets - current\ liabilities$ working capital, NWC, change in working capital
- 63 $FCFF = EBIT \times (1 - t) + Depreciation \& Ammortization - Cap.Exp. - \Delta NWC$ FCFF, free cash flow to firm, EBIT, capex, depreciation
- 64 $FCFE = FCFF - Interest \times (1 - t) + \Delta Debt$ FCFE, free cash flow to equity, net borrowing, after-tax interest
- 65 $V = \frac{FCFF_0(1+g)}{WACC - g}$ firm value, enterprise value, WACC, DCF with constant growth
- 66 $P = \frac{FCFE_0(1+g)}{r - g}$ equity value, FCFE valuation, cost of equity
- 67 $(1 + f_n) = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$ forward rate, implied forward, spot rates, term structure, yield curve
- 68 $(1 + y_n)^n = (1 + y_{n-1})^{n-1} (1 + f_n)$ long-term yield from forward rates, rolling over
- 69 $1 + y_n = [(1 + r_1)(1 + f_2)(1 + f_3) \dots (1 + f_n)]^{1/n}$ yield as average of forward rates, expectations hypothesis
- 70 $\Delta P/P = -D \times [\Delta(1+y) / (1+y)]$ bond price change using Macaulay duration
- $D = \text{Macaulay's duration}$
- 71 $D = \sum_{t=1}^T t \times w_t$ Macaulay duration, weighted average time to cash flows, effective maturity
- 72 $w_t = \frac{CF_t / (1+y)^t}{P}$ duration weights, PV of each cash flow / price

D^* = Modified duration

73 $D^* = D / (1+y)$

modified duration, convert Macaulay

74 $\frac{\Delta P}{P} = -(D^*) \Delta y$

interest rate risk, price sensitivity, yield changes by x bp, % price change

75 $Convexity = \frac{1}{P \times (1+y)^2} \sum_{t=1}^T \left[\frac{CF_t}{(1+y)^t} (t^2 + t) \right]$

convexity, curvature of price-yield curve

76 $\frac{\Delta P}{P} = -D^* \times \Delta y + \frac{1}{2} [Convexity \times (\Delta y)^2]$

duration plus convexity, large yield change, more accurate price estimate